Week 1 Lecture Review Notes

Introduction to Algebra

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1 The Basics

1.1 Numbers

Integers are whole numbers that we use for counting, and their opposites:

$$\ldots, -4, -3, -2, -1,\ 0,\ 1,\ 2,\ 3,\ 4, \ldots$$

Numbers greater than 0 are **positive** and numbers less than 0 are **negative**

0 is neither negative or positive. It is considered **nonnegative** or **nonpositive**. Sometimes it is refered to as **neutral** as well, however that is an unprofessional way of speaking of 0.

When we divide one integer by the other we get a **rational number**. Rational numbers can be written as $\frac{a}{b}$ where a and b are integers. **Irrational numbers** are numbers that can not be expressed in $\frac{a}{b}$. Irrational numbers can be numbers like π or e, where it cannot be expressed in terms of dividing two integers.

1.2 Order of Operations

There are **four** rules to remember while evaluating an expression
1) If there are parenthesis in an expression evaluate the parenthesis first and work inside out.
2) Perform all exponentiations.
3) Perform all multiplications and divisions from left to right.
4) Perform all additions and subtractions from left to right.
There is a way to remember these rules with the acronym **PEMDAS**.

Remember that this acronym is slightly wrong. Multiplication and Division can be done in any order as long as it is from left to right. The same rules apply for subtraction and addition.

1.3 When Does Order Matter?

Addition and multiplication are **commutative** which means

$$a + b = b + a \quad \text{and} \quad ab = ba$$

They are also both **associative** which means

$$a + (b + c) = (a + b) + c \quad \text{and} \quad a(bc) = (ab)c$$

Remember that subtraction and division are **not** commutative or associative.

1.4 Distribution and Factoring

For any three numbers a, b, and c, the **distributive property** states that

$$a(b + c) = ab + ac$$

Factoring is the opposite process of the distributive property. When factoring we take a common factor out of each term in a sum and write the result as this factor times a simpler sum.

$$ab + ac = a(b + c)$$

1.5 Simplifying Fractions

If the numerator and denominator of a fraction have a common factor, then that factor can be cancelled out from the numerator and denominator.
Cancelling only works when the denominator and numerator are products. We cannot cancel a term of a sum in either the numerator or denominator. The following example is not valid.

$$\frac{3+9}{2+9} \neq \frac{3}{2}$$

1.6 Properties of Equality

- The **symetric property of equality** tells us that if $a = b$, *then* $b = a$

- The **transitive property of equality** tells us that if $a = b$ and $b = c$, then $a = c$.

- If c is any number and $a = b$, then all of the following equations are true:

$$a + c = b + c \tag{1}$$

$$a - c = b - c \tag{2}$$

$$ac = bc \tag{3}$$

$$\frac{a}{c} = \frac{b}{c} \tag{4}$$

Remember that we cannot have $c = 0$ for the last equation

- If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$
- $a = b$ and $c = d$, then the following expressions are true:

$$a + c = b + d \tag{5}$$

$$a - c = b - d \tag{6}$$

$$ac = bd \tag{7}$$

2 All about X

2.1 Expressions with X

'X' is a **variable**. This means that the value of 'X' is a placeholder. Remember that the variable can be any letter.

2.2 Arithmetic with X

Note: Do NOT take these concepts very casually. We will be getting into much harder problems and concepts in this course later on. Every single concept that we will be learning will be reinforcing the basics. Great understanding of the basics can help you solve very difficult problems.